

## SURFACE AREA AND VOLUME

In this unit, we will learn to find the surface area and volume of the following three-dimensional solids:

1. Prisms
2. Pyramids
3. Cylinders
4. Cones

It is assumed that the reader has basic knowledge of the above solids and their properties from the previous unit entitled “Solids, Nets and Cross Sections.”

Some basic terminology for this section can be found below.

### Lateral Area:

The lateral area of a prism or pyramid is the combined area of its lateral faces. Similarly, the lateral area of a cylinder or cone is the area of its lateral surface.

*Examples: Given a bedroom in the shape of a rectangular prism, the lateral area is the area of the four walls. Given a cylindrical soup can, the lateral area is the area of the can's label.*

### Total Surface Area:

The total surface area of a prism or pyramid is the combined area of its lateral faces and its base(s). Similarly, the total surface area of a cylinder or cone is the combined area of its lateral surface and its base(s). The total surface area of a solid can also be thought of as the area of the solid's net.

*Examples: Given a bedroom in the shape of a rectangular prism, the total surface area is the area of the four walls plus the area of the ceiling and the floor. Given a cylindrical soup can, the total surface area is the area of the can's label plus the combined area of the top and bottom of the can.*

### Volume:

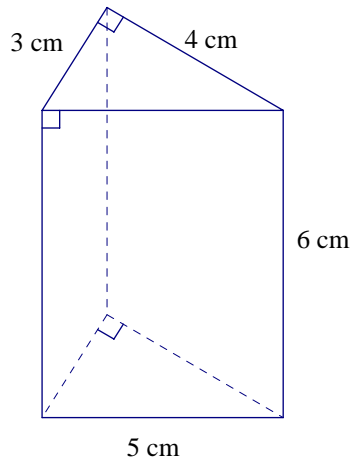
The volume of a solid is the number of cubic units which fit inside the solid.

*Example: Given a cylindrical soup can, the volume is the amount of soup which can be contained inside of the can.*

## Surface Area & Volume of a Prism

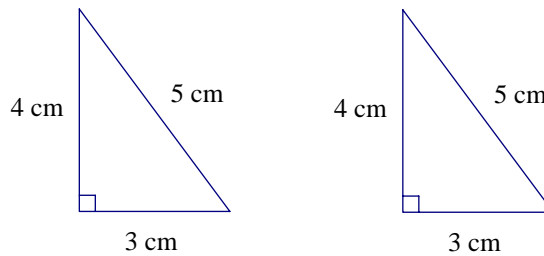
### Surface Area of a Prism

Suppose that we want to find the lateral area and total surface area of the following right triangular prism:

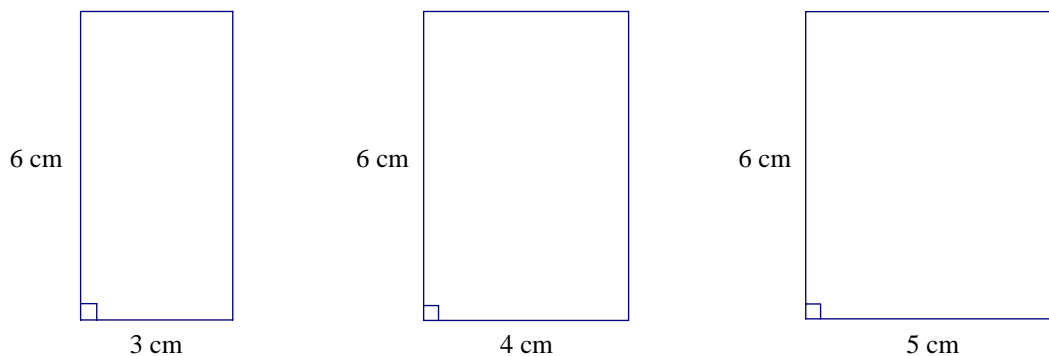


The bases of this prism are right triangles, and the lateral faces are rectangles, as shown below.

### Bases



### Lateral Faces



The lateral area of the triangular prism is the sum of the areas of the lateral faces; i.e. the sum of the areas of the three rectangles.

$$\text{Lateral Area} = 3(6) + 4(6) + 5(6) = 18 + 24 + 30 = 72 \text{ cm}^2$$

The total surface area of the triangular prism is the lateral area plus the area of the two bases.

$$\begin{aligned} \text{Total Surface Area} &= \text{Lateral Area} + \text{Area of 2 Bases} \\ &= 72 + \frac{1}{2}(3)(4) + \frac{1}{2}(3)(4) \\ &= 72 + 6 + 6 \\ &= 84 \text{ cm}^2 \end{aligned}$$

There is a bit of a shortcut for finding the lateral area of a prism. In the equation for the lateral area, notice that we can factor out a 6:

$$\text{Lateral Area} = 3(6) + 4(6) + 5(6) = (3 + 4 + 5) \cdot 6 \text{ cm}^2$$

In the formula above,  $(3 + 4 + 5)$  represents the perimeter of the base, and 6 represents the height.

The general formulas for the lateral area and total surface area of a right prism are shown below.

#### Lateral Area of a Right Prism

The lateral area  $L$  of a right prism is represented by the formula:

$$L = Ph,$$

where  $P$  represents the perimeter of a base and  $h$  represents the height of the prism.

#### Total Surface Area of a Right Prism

The total surface area  $T$  of a right prism is represented by the formula:

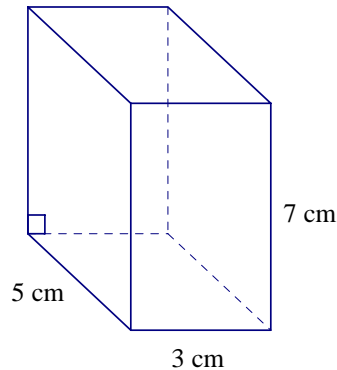
$$T = L + 2B,$$

where  $L$  represents the lateral area of the prism and  $B$  represents the area of a base.

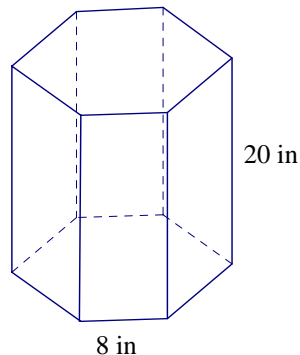
### Examples

Answer the following. Be sure to include units. (Figures may not be drawn to scale.)

1. The following figure is a rectangular right prism. In a rectangular right prism, any set of opposite faces can be considered to be the bases (since the bases of a prism are defined to be the polygonal faces that lie in parallel planes).



- a) Find the lateral area and the total surface area of the prism, assuming that the 3x5 cm rectangles are the bases.
  - b) Find the lateral area and the total surface area of the prism, assuming that the 3x7 cm rectangles are the bases.
2. The following figure is a regular right prism. Find the lateral area and the total surface area of the prism.



### Solutions:

1. a) Assuming that the 3x5 cm rectangles are the bases:  
The perimeter  $P$  of a base is  $P = 3 + 5 + 3 + 5 = 16$  cm. The height  $h$  of the prism is 7 cm.  
The lateral area of the prism is  $L = Ph = 16(7) = 112$  cm<sup>2</sup>.  
The area of a base is  $B = 3(5) = 15$  cm<sup>2</sup>.  
The total surface area is  $T = L + 2B = 112 + 2(15) = 142$  cm<sup>2</sup>.

- b) Assuming that the 3x7 cm rectangles are the bases:  
 The perimeter  $P$  of a base is  $P = 3 + 7 + 3 + 7 = 20$  cm. The height  $h$  of the prism is 5 cm.  
 The lateral area of the prism is  $L = Ph = 20(5) = 100$  cm<sup>2</sup>.  
 The area of a base is  $B = 3(7) = 21$  cm<sup>2</sup>.  
 The total surface area is  $T = L + 2B = 100 + 2(21) = 142$  cm<sup>2</sup>.

2. The bases of the prism are regular hexagons.  
 The perimeter  $P$  of a base is  $P = 6(8) = 48$  in<sup>2</sup>. The height  $h$  of the prism is 20 in.  
 The lateral area of the prism is  $L = Ph = 48(20) = 960$  in<sup>2</sup>.

To find the area  $B$  of the base, we must use the formula  $B = \frac{1}{2}aP$  (the formula for the area of a regular polygon). The apothem,  $a$ , is  $4\sqrt{3}$  in. (For a detailed description of how to find the apothem and area of a regular polygon, refer to the unit entitled “Perimeter and Area” and then refer to the last section in the unit, entitled “Area of a Regular Polygon.”) Since the perimeter  $P$  of the regular hexagon is 48 in,  $B = \frac{1}{2}aP = \frac{1}{2}(4\sqrt{3})(48) = 96\sqrt{3}$  in<sup>2</sup>.

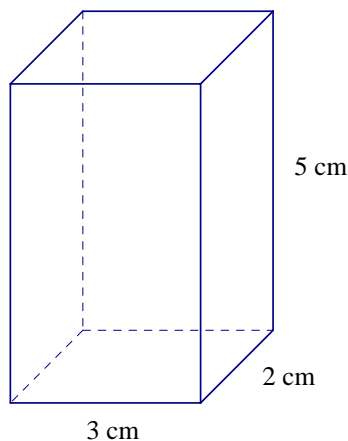
Therefore, the total surface area of the regular hexagonal prism is

$$T = L + 2B = 960 + 2(96\sqrt{3}) = (960 + 192\sqrt{3}) \text{ in}^2.$$

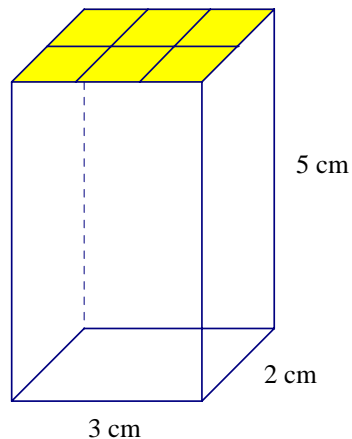
*Note: The above terms are not like terms and cannot be combined, since one term contains a radical symbol and the other does not.*

### Volume of a Prism

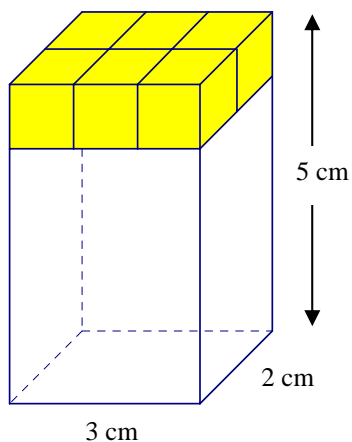
We are now going to informally develop the formula for the volume of a prism. Consider the following right rectangular prism.



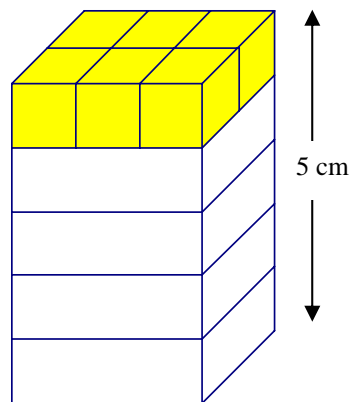
First, we find the area of the base. We will call this area  $B$ .  $B = 3(2) = 6 \text{ cm}^2$ . The six square units of area are drawn on one of the bases in the diagram at the right.



In the diagram at the right, each square unit of area on the base has now been used to draw a cubic unit of volume. Six cubes are contained in this first 'layer' of the prism which is one-centimeter thick.



Since the prism has a height of 5 cm, there are five layers which are each one-centimeter thick. Since each layer contains six cubic units, the volume  $V$  of the prism is  $V = 6(5) = 30 \text{ cm}^3$ .



The above method leads us to a volume formula that can be used for any prism.

### Volume of a Prism

The volume  $V$  of a prism is represented by the formula:

$$V = Bh,$$

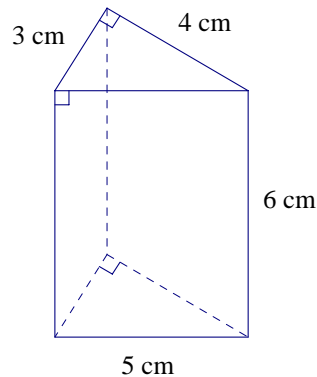
where  $B$  represents the area of a base and  $h$  represents the height of the prism.

*\*Note that this formula works for both right and oblique prisms.*

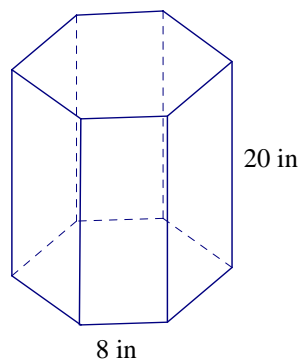
### Examples

Answer the following. Be sure to include units. (Figures may not be drawn to scale.)

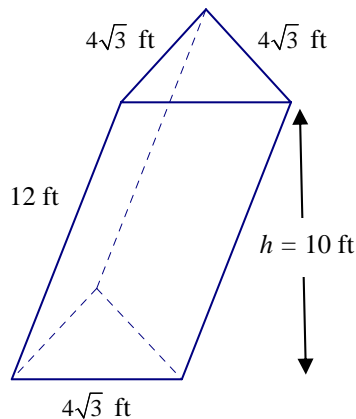
1. Find the volume of the following right triangular prism.



2. Find the volume of the following regular right prism.



3. Find the volume of the following regular oblique prism.



Solutions:

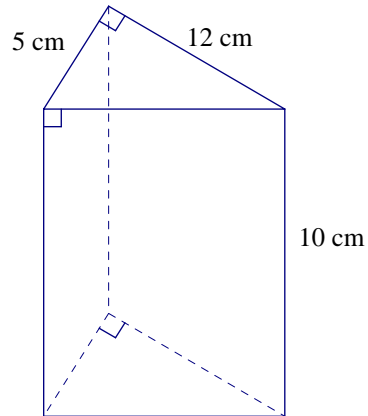
- The base of the prism is a right triangle with area  $B = \frac{1}{2}(4)(3) = 6 \text{ cm}^2$ .  
The height  $h$  of the prism is  $h = 6$  cm.  
Therefore, the volume  $V$  is  $V = Bh = 6(6) = 36 \text{ cm}^3$ .
- To find the area  $B$  of the base, we must first use the formula  $B = \frac{1}{2}aP$ . (the formula for the area of a regular polygon). The apothem,  $a$ , has a length of  $4\sqrt{3}$  in. (For a detailed description of how to find the apothem and area of a regular polygon, refer to the unit entitled "Perimeter and Area" and then refer to the last section in the unit, entitled "Area of a Regular Polygon.") The perimeter of the regular hexagon is  $P = 6(8) = 48$  in.  
So  $B = \frac{1}{2}aP = \frac{1}{2}(4\sqrt{3})(48) = 96\sqrt{3} \text{ in}^2$ .  
The height  $h$  of the prism is  $20$  in.  
Therefore, the volume  $V$  is  $V = Bh = 96\sqrt{3}(20) = 1920\sqrt{3} \text{ in}^3$ .
- To find the area  $B$  of the base, we must first use the formula  $B = \frac{1}{2}aP$ . (the formula for the area of a regular polygon). The apothem,  $a$ , has a length of  $2$  ft. (For a detailed description of how to find the apothem and area of a regular polygon, refer to the unit entitled "Perimeter and Area" and then refer to the last section in the unit, entitled "Area of a Regular Polygon.") The perimeter  $P$  of the equilateral triangle is  $P = 3(4\sqrt{3}) = 12\sqrt{3}$  ft.  
So  $B = \frac{1}{2}aP = \frac{1}{2}(2)(12\sqrt{3}) = 12\sqrt{3} \text{ ft}^2$ .  
The height  $h$  of the prism is  $10$  ft.  
Therefore, the volume  $V$  is  $V = Bh = 12\sqrt{3}(10) = 120\sqrt{3} \text{ ft}^3$ .  
(Notice that the  $12$  ft measure was not needed at all to find the volume.)



### Exercises

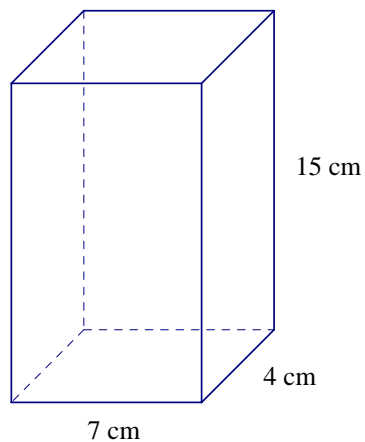
Answer the following. Be sure to include units. (Figures may not be drawn to scale.)

1. Given the following right triangular prism,



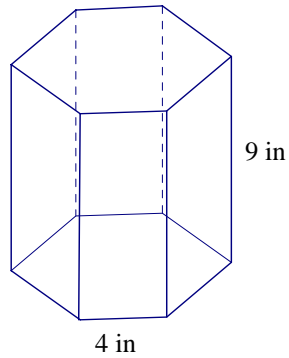
- Find the missing side length of the base.
- Find the lateral area.
- Find the total surface area.
- Find the volume.

2. Given the following right rectangular prism,



- Find the lateral area. (Assume that the bases are the 4x7 cm rectangles.)
- Find the total surface area.
- Find the volume.

3. Given the following right regular hexagonal prism,



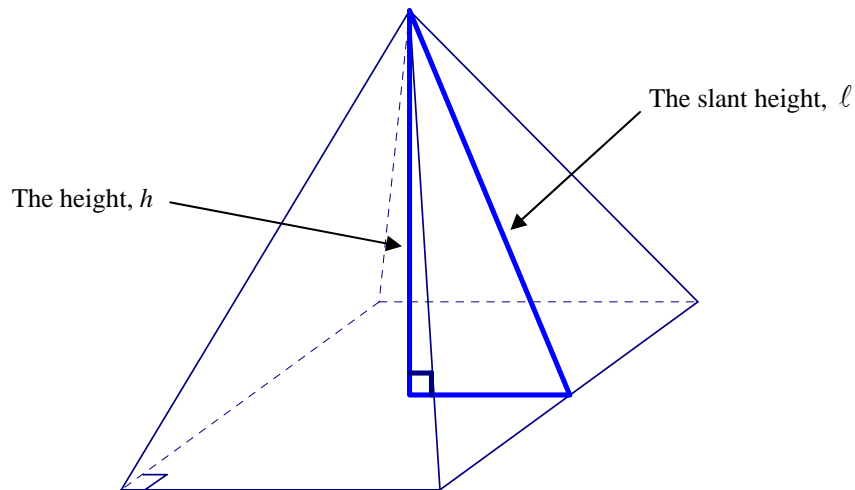
- Find the lateral area.
  - Find the total surface area.
  - Find the volume.
4. A rectangular prism with a square base has a height of 7 m and a volume of  $175 \text{ m}^3$ .
- Find the dimensions of the square base.
  - Find the lateral area.
  - Find the total surface area.
5. A rectangular prism has a square base, and the side of the square base is 3 in. If the volume of the prism is  $108 \text{ in}^3$ ,
- Find the height of the prism.
  - Find the lateral area.
  - Find the total surface area.
6. A cube has a lateral area of  $144 \text{ cm}^2$ . Find the length of an edge.
7. A cube has a total surface area of  $96 \text{ m}^2$ . Find the length of an edge.
8. A cube has a volume of  $343 \text{ cm}^3$ . Find the length of an edge.

## Surface Area & Volume of a Pyramid

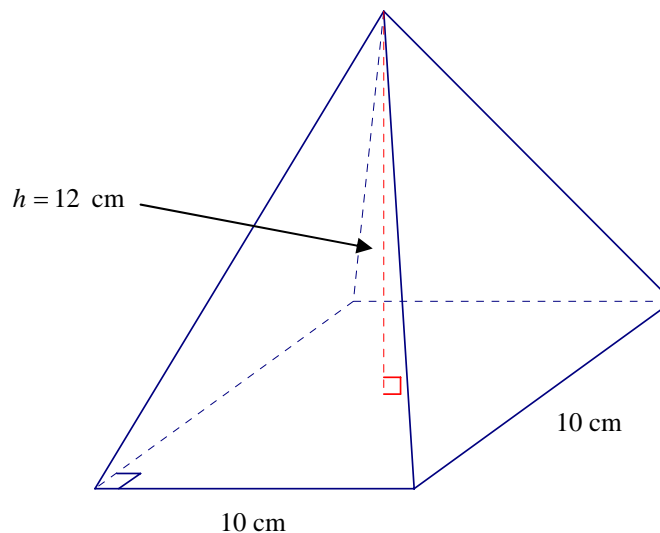
### Surface Area of a Pyramid

When finding the surface area of a pyramid, we will limit our discussion to the study of regular right pyramids. (A right pyramid is a pyramid in which the apex is directly above the center of the base.)

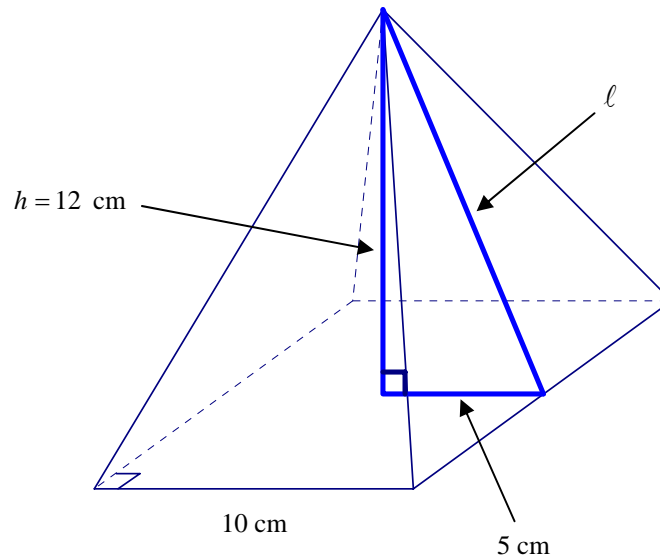
First, let us define and illustrate a few important terms. The height of a right pyramid is the distance from the apex of the pyramid to the center of its base. The height is abbreviated as  $h$ . The slant height of a regular right pyramid is the distance from the apex to the midpoint of the side of a base; i.e. the height of one of the lateral triangular faces. The slant height is abbreviated as  $\ell$ . (In a regular right pyramid, all slant heights are congruent, since all of the lateral faces are congruent.)



Consider the following right pyramid with a square base. We want to find its lateral area and total surface area:



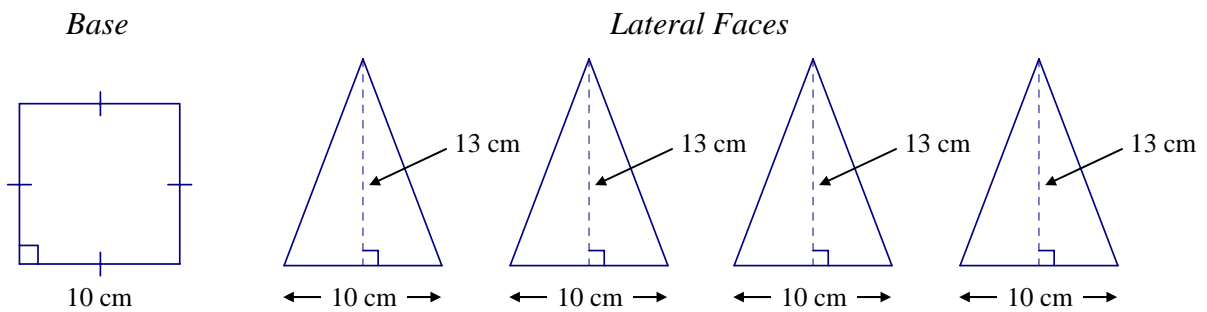
Notice that the slant height is not given in the above figure. We will first draw a right triangle within the pyramid and find the slant height,  $\ell$ , as shown below.



Since the altitude of the triangle intersects the center of the base, we know that the horizontal leg of the right triangle is 5 cm (half of the 10 cm side of the square base). We then use the Pythagorean Theorem to find  $\ell$ :

$$\begin{aligned} 5^2 + 12^2 &= \ell^2 \\ 25 + 144 &= \ell^2 \\ 169 &= \ell^2 \\ \ell &= 13 \text{ cm} \end{aligned}$$

The base of this pyramid is a square, and the lateral faces are triangles, as shown below. Notice that the height of each triangular lateral face is 13 cm, *not* 12 cm.



The lateral area of the square pyramid is the sum of the areas of the lateral faces; i.e. the sum of the areas of the four congruent triangles.

$$\begin{aligned} \text{Lateral Area} &= \frac{1}{2}(10)(13) + \frac{1}{2}(10)(13) + \frac{1}{2}(10)(13) + \frac{1}{2}(10)(13) \\ &= 65 + 65 + 65 + 65 = 260 \text{ cm}^2 \end{aligned}$$

The lateral area of the pyramid is  $260 \text{ cm}^2$ .

The total surface area of the pyramid is the lateral area plus the area of the base.

$$\begin{aligned} \text{Total Surface Area} &= \text{Lateral Area} + \text{Area of Base} \\ &= 260 + 10^2 \\ &= 260 + 100 \\ &= 360 \text{ cm}^2 \end{aligned}$$

The total surface area of the pyramid is  $360 \text{ cm}^2$ .

There is a shortcut for finding the lateral area of a regular right pyramid. In the equation for the lateral area, we will factor out  $\frac{1}{2}(13)$ :

$$\begin{aligned} \text{Lateral Area} &= \frac{1}{2}(10)(13) + \frac{1}{2}(10)(13) + \frac{1}{2}(10)(13) + \frac{1}{2}(10)(13) \\ &= \frac{1}{2}(13)(10 + 10 + 10 + 10) \end{aligned}$$

In the formula above,  $(10 + 10 + 10 + 10)$  represents the perimeter of the base,  $P$ , and 13 represents the slant height,  $\ell$ .

The general formulas for the lateral area and total surface area of a regular right pyramid are shown below.

#### Lateral Area of a Regular Right Pyramid

The lateral area  $L$  of a regular right pyramid is represented by the formula:

$$L = \frac{1}{2} \ell P,$$

where  $P$  represents the perimeter of the base and  $\ell$  represents the slant height of the pyramid.

### Total Surface Area of a Regular Right Pyramid

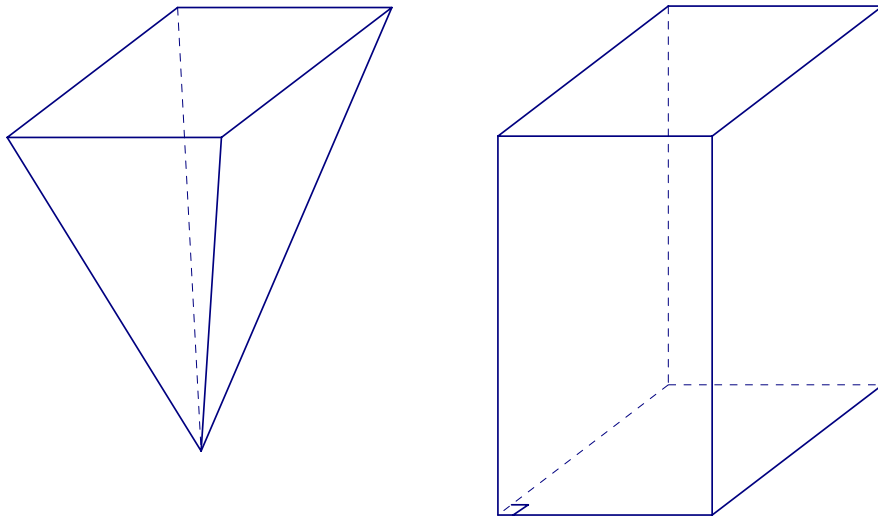
The total surface area  $T$  of a regular right pyramid is represented by the formula:

$$T = L + B,$$

where  $L$  represents the lateral area of the pyramid and  $B$  represents the area of the base.

### Volume of a Pyramid

Suppose that we want to compare the volume of the following solids. The prism and the pyramid illustrated below have the same base and the same height.



Suppose that we decide to fill the pyramid with sand and then pour it into the prism. We then continue this process until the prism is completely full of sand. How many pyramids of sand do you think would fit into the prism?

It turns out that we would need to fill the pyramid three times. In other words, if a pyramid and the prism both have the same base and height, the volume of the pyramid is one-third the volume of the prism. The proof of this formula is beyond the scope of this course and will not be shown here, but the formula can be found below.

### Volume of a Pyramid

The volume  $V$  of a pyramid is represented by the formula:

$$V = \frac{1}{3}Bh,$$

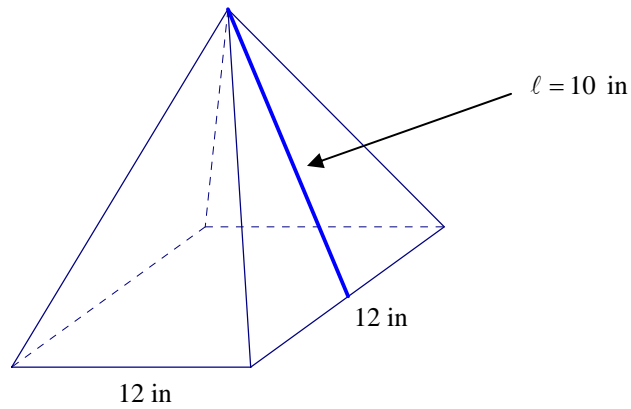
where  $B$  represents the area of the base and  $h$  represents the height of the pyramid.

*Note: This formula applies to any pyramid, even if the base is not regular, and if the pyramid is not a right pyramid.*

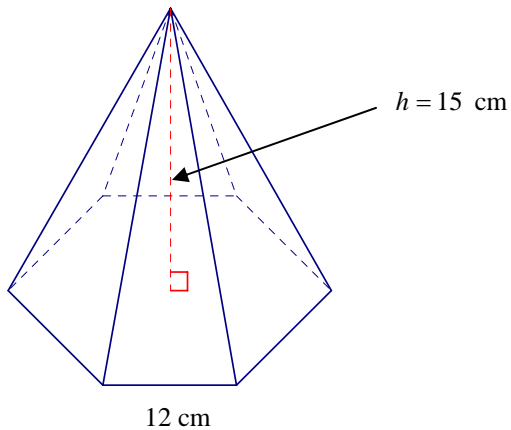
### **Examples**

Find the volume of each of the following regular right pyramids. Be sure to include units. (Figures may not be drawn to scale.)

1.

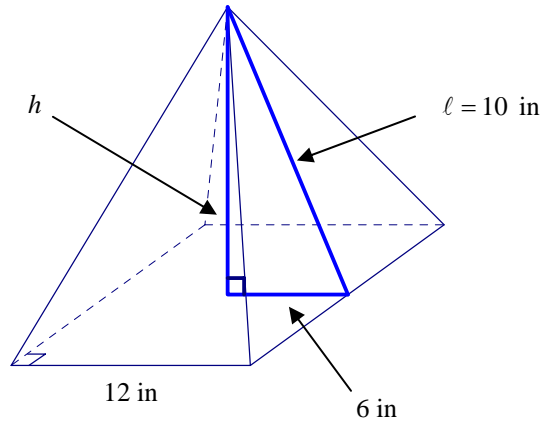


2.



Solutions:

1. We first need to find the height,  $h$ , of the pyramid. We draw an altitude to the base of the pyramid and form a right triangle as shown below. We can then use the Pythagorean Theorem to find  $h$ .



$$\begin{aligned}6^2 + h^2 &= 10^2 \\36 + h^2 &= 100 \\h^2 &= 64 \\h &= 8 \text{ in.}\end{aligned}$$

The base of the pyramid is a square with area  $B = 12^2 = 144 \text{ in}^2$ .

The height  $h$  of the pyramid is  $h = 8$  in.

Therefore, the volume  $V$  is  $V = \frac{1}{3}Bh = \frac{1}{3}(144)(8) = 384 \text{ in}^3$ .

(Notice that the 10 in measure was not needed at all to find the volume. This would, however, be important in determining the lateral area or total surface area.)

2. To find the area  $B$  of the base, we must first use the formula  $B = \frac{1}{2}aP$ . (the formula for the area of a regular polygon). The apothem,  $a$ , has a length of  $6\sqrt{3}$  in. (For a detailed description of how to find the apothem and area of a regular polygon, refer to the unit entitled “Perimeter and Area” and then refer to the last section in the unit, entitled “Area of a Regular Polygon.”) The perimeter of the regular hexagon is  $P = 6(12) = 72$  cm.

$$\text{So } B = \frac{1}{2}aP = \frac{1}{2}(6\sqrt{3})(72) = 216\sqrt{3} \text{ cm}^2.$$

The height  $h$  of the prism is 15 in. Therefore, the volume  $V$  is

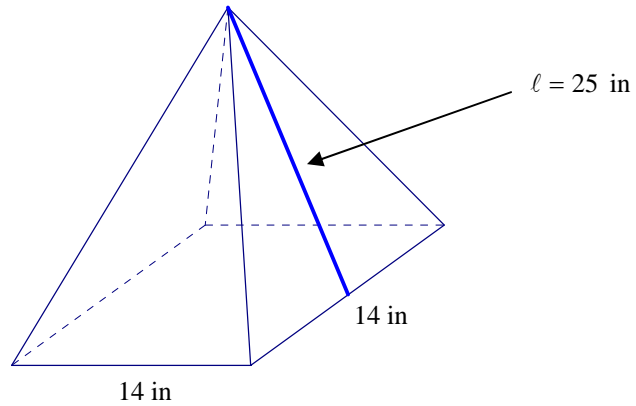
$$V = \frac{1}{3}Bh = \frac{1}{3}(216\sqrt{3})(15) = \frac{1}{3}(3240\sqrt{3}) = 1080\sqrt{3} \text{ cm}^3.$$



### Exercises

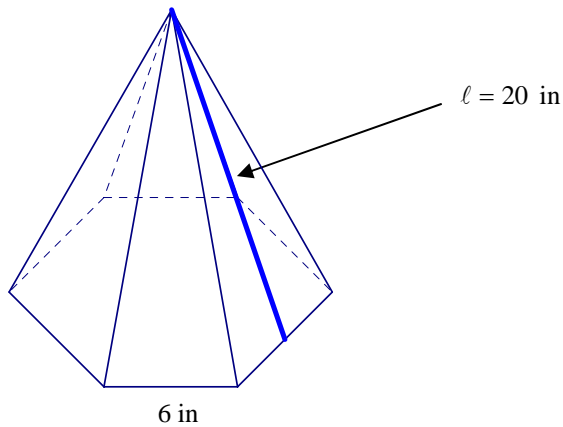
Answer the following. Be sure to include units. (Figures may not be drawn to scale.)

1. Given the following right pyramid with a square base,



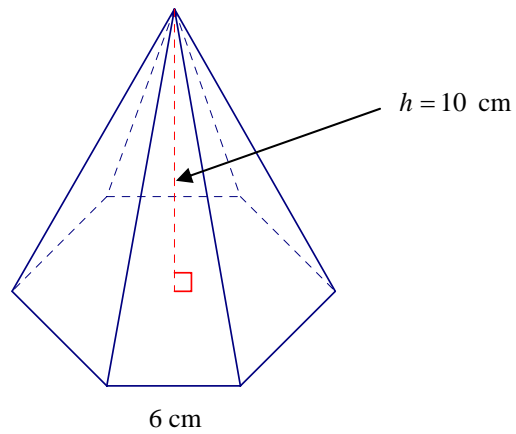
- Find the lateral area.
- Find the total surface area.
- Find the volume.

2. Given the following regular right pyramid,



- Find the lateral area.
- Find the total surface area.

3. Find the volume of the following regular right pyramid.



4. A right pyramid with a square base has a height of 10 cm and a volume of  $120 \text{ cm}^3$ . Find the dimensions of the base.
5. A right pyramid with a square base has a slant height of 8 in and a lateral area of  $80 \text{ in}^2$ . Find the dimensions of the base.

## Surface Area & Volume of a Cylinder

A cylinder can be likened to a prism with a many-sided base. Because of this fact, the formulas for a cylinder are relatively easy to establish. When a formula for a prism calls for the *perimeter* of the base, we replace perimeter with *circumference*, as shown below.

	<u>Formulas for a Prism</u>	<u>Formulas for a Cylinder</u>
Lateral Area	$L = Ph$	$L = Ch$
Total Surface Area	$T = L + 2B$	$T = L + 2B$
Volume	$V = Bh$	$V = Bh$

Since the bases of a cylinder are circles, we use the formula  $B = \pi r^2$  for the area of a circle and  $C = \pi d = 2\pi r$  for the circumference of a circle.

The formulas for the lateral area, total surface area and volume of a cylinder can be found below.

### Lateral Area of a Right Cylinder

The lateral area  $L$  of a right circular cylinder is represented by the formula:

$$L = Ch, \text{ where } C = \pi d = 2\pi r.$$

$C$  represents the circumference of a base,  $h$  represents the height of the cylinder, and  $d$  and  $r$  represent the diameter and radius of a base, respectively.

### Total Surface Area of a Right Cylinder

The total surface area  $T$  of a right circular cylinder is represented by the formula:

$$T = L + 2B, \text{ where } B = \pi r^2.$$

$L$  represents the lateral area of the cylinder,  $B$  represents the area of a base, and  $r$  represents the radius of a base.

### Volume of a Cylinder

The volume  $V$  of a right circular cylinder is represented by the formula:

$$V = Bh, \text{ where } B = \pi r^2.$$

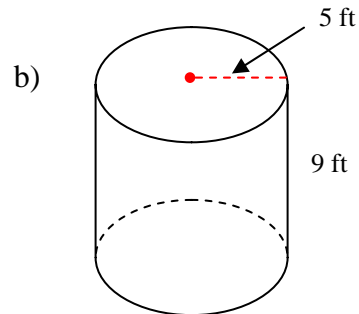
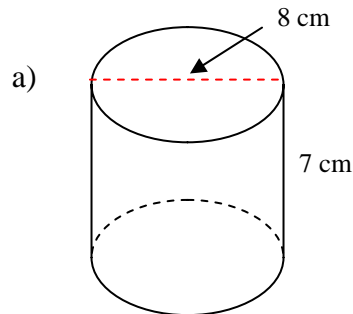
$B$  represents the area of a base,  $h$  represents the height of the cylinder, and  $r$  represents the radius of a base.

*\*Note that this formula works for both right and oblique cylinders.*

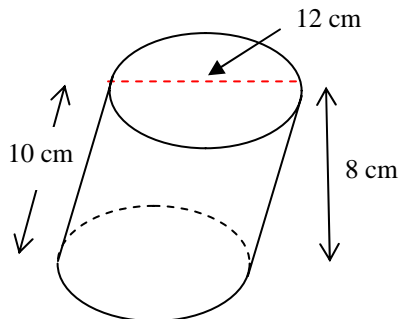
### **Examples**

Answer the following. Be sure to include units. (Figures may not be drawn to scale.)

1. Find the lateral area, total surface area, and volume of each of the following right circular cylinders.



2. Find the volume of the following oblique circular cylinder.



Solutions:

1. a) From the diagram of the cylinder, we can see that  $d = 8$  cm and  $h = 7$  cm .

We first find the circumference,  $C$ :

$$C = \pi d = \pi \cdot 8 = 8\pi \text{ cm}$$

We can now find the lateral area,  $L$ :

$$L = Ch = 8\pi \cdot 7 = 56\pi \text{ cm}^2$$

Since  $d = 8$  cm, then  $r = 4$  cm. We then compute the area of the base,  $B$ :

$$B = \pi r^2 = \pi \cdot 4^2 = 16\pi \text{ cm}^2$$

Next, we compute the total surface area,  $T$ :

$$T = L + 2B = 56\pi + 2 \cdot 16\pi = 56\pi + 32\pi = 88\pi \text{ cm}^2$$

Finally, we compute the volume,  $V$ :

$$V = Bh = 16\pi \cdot 7 = 112\pi \text{ cm}^3$$

- b) From the diagram of the cylinder, we can see that  $r = 5$  ft and  $h = 9$  ft .

We first find the circumference,  $C$ :

$$C = 2\pi r = 2\pi \cdot 5 = 10\pi \text{ ft}$$

We can now find the lateral area,  $L$ :

$$L = Ch = 10\pi \cdot 9 = 90\pi \text{ ft}^2$$

We then compute the area of the base,  $B$ :

$$B = \pi r^2 = \pi \cdot 5^2 = 25\pi \text{ ft}^2$$

Next, we compute the total surface area,  $T$ :

$$T = L + 2B = 90\pi + 2 \cdot 25\pi = 90\pi + 50\pi = 140\pi \text{ ft}^2$$

Finally, we compute the volume,  $V$ :

$$V = Bh = 25\pi \cdot 9 = 225\pi \text{ ft}^3$$

2. From the diagram of the cylinder, we can see that  $d = 12$  cm and  $h = 8$  cm .

Since  $d = 12$  cm, then  $r = 6$  cm. We then compute the area of the base,  $B$ :

$$B = \pi r^2 = \pi \cdot 6^2 = 36\pi \text{ cm}^2$$

We can then find the volume,  $V$ :

$$V = Bh = 36\pi \cdot 8 = 288\pi \text{ cm}^3$$

**Exercises**

Answer the following. Be sure to include units.

1. A right circular cylinder has a radius of 10 in and a height of 4 in.
  - a) Find the lateral area.
  - b) Find the total surface area.
  - c) Find the volume.

2. A right circular cylinder has a diameter of 14 in and a height of 9 in.
  - a) Find the lateral area.
  - b) Find the total surface area.
  - c) Find the volume.
  
3. A right circular cylinder has a diameter of 5 m and a height of 6 m.
  - a) Find the lateral area.
  - b) Find the total surface area.
  - c) Find the volume.
  
4. A right circular cylinder has a height of 5 in and a volume of  $245\pi \text{ in}^3$ .
  - a) Find the radius.
  - b) Find the lateral area.
  - c) Find the total surface area.
  
5. A right circular cylinder has a diameter of 16 cm and a lateral area of  $192\pi \text{ cm}^2$ .
  - a) Find the height
  - b) Find the total surface area.
  - c) Find the volume.

## Surface Area & Volume of a Cone

Just as the formulas for a cylinder were similar to those of a prism, the formulas for a cone are similar to those of a pyramid. When a formula for a pyramid calls for the *perimeter* of the base, we replace perimeter with *circumference*, as shown below.

	<u>Formulas for a Pyramid</u>	<u>Formulas for a Cone</u>
Lateral Area	$L = \frac{1}{2} \ell P$	$L = \frac{1}{2} \ell C$
Total Surface Area	$T = L + B$	$T = L + B$
Volume	$V = \frac{1}{3} Bh$	$V = \frac{1}{3} Bh$

Since the base of a cone is a circle, we use the formula  $B = \pi r^2$  for the area of the circle and  $C = \pi d = 2\pi r$  for the circumference of the circle.

The formulas for the lateral area, total surface area and volume of a cone can be found below.

### Lateral Area of a Right Cone

The lateral area  $L$  of a right circular cone is represented by the formula:

$$L = \frac{1}{2} \ell C, \text{ where } C = \pi d = 2\pi r.$$

$C$  represents the circumference of the base,  $\ell$  represents the slant height of the cone, and  $d$  and  $r$  represent the diameter and radius of the base, respectively.

### Total Surface Area of a Right Cone

The total surface area  $T$  of a right circular cone is represented by the formula:

$$T = L + B, \text{ where } B = \pi r^2.$$

$L$  represents the lateral area of the cone,  $B$  represents the area of a base, and  $r$  represents the radius of a base.

### Volume of a Cone

The volume  $V$  of a right circular cone is represented by the formula:

$$V = \frac{1}{3}Bh, \text{ where } B = \pi r^2$$

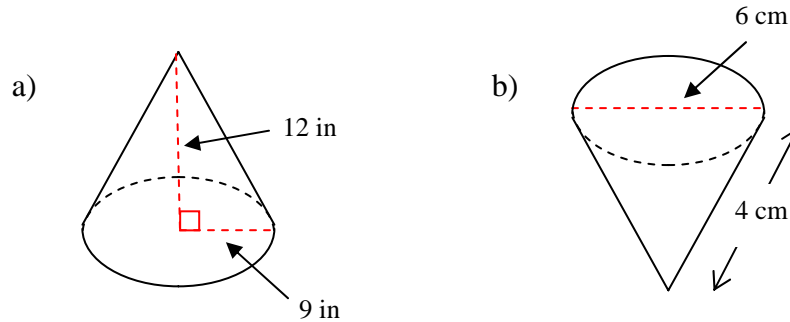
where  $B$  represents the area of the base,  $h$  represents the height of the cone, and  $r$  represents the radius of the base.

*Note: This formula applies to any circular cone, even if the cone is not a right circular cone.*

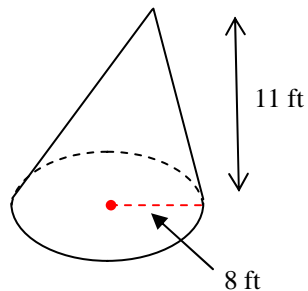
### **Examples**

Answer the following. Be sure to include units. (Figures may not be drawn to scale.)

1. Find the lateral area, total surface area, and volume of each of the following right circular cones.



2. Find the volume of the following oblique circular cylinder.





Solutions:

1. a) From the diagram of the cone, we can see that  $r = 9$  in and  $h = 12$  in .  
Notice that the slant height  $\ell$  is not given, but we can find it using the Pythagorean Theorem:

$$9^2 + 12^2 = \ell^2$$

$$81 + 144 = \ell^2$$

$$225 = \ell^2$$

$$\ell = 15 \text{ in}$$

Next, we find the circumference,  $C$ :

$$C = 2\pi r = 2\pi \cdot 9 = 18\pi \text{ in}$$

We can now find the lateral area,  $L$ :

$$L = \frac{1}{2} \ell C = \frac{1}{2} (15)(18\pi) = 135\pi \text{ in}^2$$

We then compute the area of the base,  $B$ :

$$B = \pi r^2 = \pi \cdot 9^2 = 81\pi \text{ in}^2$$

Next, we compute the total surface area,  $T$ :

$$T = L + B = 135\pi + 81\pi = 216\pi \text{ in}^2$$

Finally, we compute the volume,  $V$ :

$$V = \frac{1}{3} Bh = \frac{1}{3} (81\pi)(12) = 324\pi \text{ in}^3$$

- b) From the diagram of the cone, we can see that  $d = 6$  cm and  $\ell = 4$  cm .  
Since  $d = 6$  cm, then  $r = 3$  cm.

Notice that the height  $h$  is not given, but we can find it using the Pythagorean Theorem:

$$3^2 + h^2 = 4^2$$

$$9 + h^2 = 16$$

$$h^2 = 7$$

$$h = \sqrt{7} \text{ cm}$$

Next, we first find the circumference,  $C$ :

$$C = \pi d = \pi \cdot 6 = 6\pi \text{ cm}$$

We can now find the lateral area,  $L$ :

$$L = \frac{1}{2} \ell C = \frac{1}{2} (4)(6\pi) = 12\pi \text{ cm}^2$$

We then compute the area of the base,  $B$ :

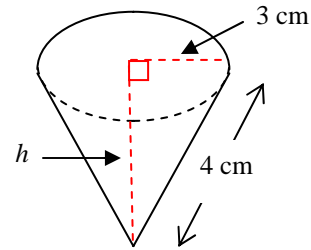
$$B = \pi r^2 = \pi \cdot 3^2 = 9\pi \text{ cm}^2$$

Next, we compute the total surface area,  $T$ :

$$T = L + B = 12\pi + 9\pi = 21\pi \text{ cm}^2$$

Finally, we compute the volume,  $V$ :

$$V = \frac{1}{3} Bh = \frac{1}{3} (9\pi)(\sqrt{7}) = 3\pi\sqrt{7} \text{ cm}^3$$



2. From the diagram of the cone, we can see that  $r = 8$  ft and  $h = 11$  ft .

We then compute the area of the base,  $B$ :

$$B = \pi r^2 = \pi \cdot 8^2 = 64\pi \text{ ft}^2$$

We can then find the volume,  $V$ :

$$V = \frac{1}{3} Bh = \frac{1}{3}(64\pi)(11) = \frac{704}{3} \pi \text{ ft}^3$$

### Exercises

Answer the following. Be sure to include units in your answer.

1. A right circular cone has a radius of 7 cm and a slant height of 25 cm.
  - a) Find the lateral area.
  - b) Find the total surface area.
  - c) Find the volume.
2. A right circular cone has a diameter of 4 ft and a height of 5 ft.
  - a) Find the lateral area.
  - b) Find the total surface area.
  - c) Find the volume.
3. A right circular cone has a radius of 5 in and a lateral area of  $35\pi \text{ in}^2$  .
  - a) Find the slant height.
  - b) Find the height.
  - c) Find the total surface area.
  - d) Find the volume.
4. A right circular cone has a diameter of 18 in and a volume of  $108\pi \text{ in}^3$  .
  - a) Find the height.
  - b) Find the slant height.
  - c) Find the lateral surface area.
  - d) Find the total surface area.